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Qualitative Characteristics
of Reasoning by
Qualitative Data

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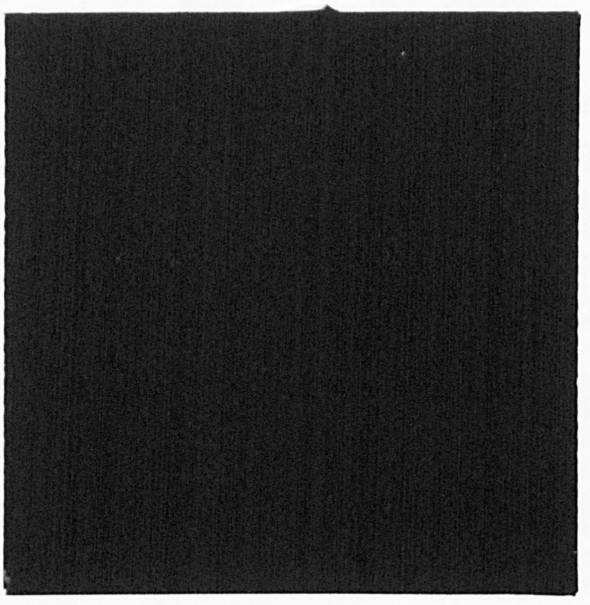

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REPRESENTING THE QUALITATIVE CHARACTERISTICS OF THE REASONING BY QUALITATIVE DATA

Two examples from the field of operational abilities: combinative and logical operations

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I would like to express my special thanks to Professor HANS-JÖRG HENNING, who first directed my attention to the possibilities of qualitative data analysis.

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1. Exactness and quality

One of the widely accepted beliefs concerning mathematics is that mathematics is the science of the quantities. From a historical point of view this may be true for the first phase of the development of mathematics, but it does not hold for its present state. During the last three centuries new disciplines have appeared in the field of mathematics, such as set theory, algebra, combinatorics, mathematical logic, graph theory, etc. In these disciplines the structures and qualities play an important role, and in this century these disciplines have become the most intensively examined areas of mathematics. Some fields appeared only in the 1940-s or 1950-s (mathematical system theory, computer science), and mathematics has recently begun to cope with one of the most interesting challenges in its history: to devise exact concepts for the characterization of "unexactness", as the theory of fuzzy systems and fuzzy subsets tries to do.

These changes in mathematics were caused mainly by the internal logic of its development. But apart from this, there was always a close interaction between the development of mathematics and that of the natural sciences. Until about the 1920-s the main application of mathematics was "classical physics". Two of the most perfect "constructions" of that science are Newtonian mechanics and the electrodynamics of Maxwell. Both of thse use the tools of mathematical (function) analysis, differential equations.

The changes in physics in the early decades of this century produced new needs for mathematical tools, and these were no longer the mathematics of measures and quantities, but the mathematics of qualities and structures. The needs of quantum-mechanics and nuclear physics produced new disciplines (e.g. quantum logic), and other sciences currently make use of non-numerical mathematics (e.g. stereochemistry uses the graph theory, etc.).

The first efforts to put the social sciences on a "more exact" basis were made before the above-mentioned changes in the natural sciences, and accordingly they had the aim ofintroducing the idea of measurement into these fields. Until the recent reform-movement of new math, most educational systems,transmitted mainly the rudiments of numerical mathematics, and the classical view of the sciences. Thus it is not surprising that there is a far reaching association between the concepts of "scientific" and "measurable".

The development of the role of mathematics in psychology was very similar. The initial efforts were concentrated on making the things measurable and then applying quantitative methods. The unmeasurable properties seemed to be beyond the possibilities of mathematical methods, and hence the term "qualitative" for a long time meant something nonexact, or even a low grade of scientific value.

The wide use of quantitative methods is understandable.

Only the recent trends in data analysis offer a possibility to preserve the qualitative features of things, and analyse them without the necessity of quantification, but at the same time without a loss of exactness. However, apart from the lack of adequate qualitative methods, there was probably another cause of the preference for quantification: in some cases the representation of qualitative properties needs much more information than that of quantitative ones, and only the increasing capacities of computers made it possible to store and to process such information.

As a result of the rapid development of the new methods, the meaning of the expression "qualitative data analysis" has been changing. In the book of Miles and Huberman, "qualitative data" means mainly verbal information and the emphasis is placed on "qualitative analysis". In their view, "... the data concerned appear in words rather than in numbers. They may have been collected in a variety of ways (observations, interviews, extracts from documents, tape recordings), and are usually 'processed' somewhat before they are ready for use (via dictation, typing up, editing, or transcription), but they remain words, usually organized into extended text. ... We consider that analysis consists of three concurrent flows of activity: data reduction, data display, and conclusion drawing/verification." (p. 21). The most recent approaches (Henning and Rudinger, 1985; Rudinger, Chaselon, Zimmermann and Henning, 1985) use a more exact concept of qualitative data. In this formulation, qualitative data are sets, sequences or matrices of numbers which represent theoretical structures. And "qualitative analyses" are statistical methods which are adequate to handle and process them. In the present paper I use this more sophisticated concept of qualitative data.

One area of our present research efforts on the development of operational abilities is concentrated on finding appropriate ways of representing the developmental levels and processes of the operational abilities. By presenting examples from this research, I would like to illustrate the following statements:

1. Exactness and quality are not incompatible concepts in the field of psychology. Qualitative properties can also be represented and processed exactly.

2. The application of qualitative methods and the tools of non-numerical mathematics is not a consequence of our inability to quantify them (we can do this), but it is an appropriate approach to study, to describe and to model them. The complexity of the subject requires application of the methods of qualitative and structural analysis. Quantification can cause the loss of psychologically relevant information.

3. The representation of qualitative properties needs more information-storing capacity, but the use of a computer makes this possible and the scientific results make qualitative analysis worthwhile.

This paper does not intend to be a methodologist's contribution to the solution of these problems, but rather a conceptualization of the expectations, needs and possibilities of the application of the methods of qualitative data analysis to the field of ability research.

2. The examination of operational abilities

Most of the research projects investigating the development of cognitive operations have been influenced by the works of Piaget and the Geneva School. The purpose of many of these works was to produce evidence for or against the Piagetian theory (see Modgil and Modgil, 1982; Cohen, 1983). Some research applied the original clinical methods under better controlled conditions, and others tried to construct tests to measure the developmental levels of operational reasoning (e.g. Roberge, 1976; Roberge and Flexer, 1982). There are many re-formulations of the Piagetian theory that use a more precise conceptualization (Keats, Collis and Halford, 1978) or more recent approaches, such as the information-processing models of cognitive psychology (Klahr and Wallace, 1976).

The main advantage of the Piagetian approach is its deep mathematical-theoretical background and the possibility of studying the qualitative nature of reasoning. Its main weaknesses are the verbal representation of the results, the lack of further statistical analysis, and the subjectivity in the course of examination and interpretation. Most of the neo-Piagetian researchers overcame these problems, but they stuck mainly to the conventional Piagetian tasks and situations. On the other hand, the tests devised for the measurement of operational reasoning usually contain only some items (3-5 tasks) concerning logical or combinative operations, and the method of evaluation is a simple scoring of the achievements. Such tests are perhaps suitable tools for indicating the general developmental level of reasoning, and can afford quantitative data for statistical analysis, but they lack the possibility to detect fine qualitative differences and changes in the development of thinking.

In the middle of the 1970-s, a research group at Szeged University began to deal with the development of operational abilities. The efforts were focused on three systems of operations, the combinative (B. Csapó), the logical (E. Czachesz and T. Vidákovich) and the systematizing (J. Nagy) operations. One of the origins of this research was the Piagetian tradition, but from those principles we retained only the mathematical background, and the intention of studying cognitive development as a process of qualitative changes. Our aim was to combine the advantages of the qualitative approach with the test-assessment techniques. Thus, we elaborated a detailed mathematical structure and, on that basis, a test-battery for the assessment of the operational abilities. These tests were administered to large groups and afforded qualitative data about operational reasoning. These data can easily be quantified for conventional statistical analyses.

It is not possible to go into the details of that research in this paper. The basic concepts and some early results concerning the combinative operations can be found in Csapó (1985a, 1985b); and about logical operations in Csapó, Czachesz and Vidákovich (1986). In the following pages, only the methodical problems of representing qualitative characteristics of reasoning will be illustrated by presenting some examples from our current work.

During the past few years we have carried out several measurements on children of different ages (ranging from 6 to 17 years) by using these tests. Because of the lack of adequate qualitative methods and further problems discussed later, in the first phase of the research we quantified the data and used multidimensional statistical methods (see Csapó, 1987). In the present stage we are devising statistical programs for the processing of the qualitative data, and we are beginning to apply the methods afforded by the most recent developments in this field.

3. Representation and analysis of quality in combinatorial reasoning

3.1. The qualitative representation and the ways of quantification

First, with the help of some examples, I shall sow, how the qualitative nature of the combinative operations can be studied, and how the quality of reasoning can be transformed into qualitative data. I shall then deal with the possibilities of quantification. In this way we can compare the positive and negative features of the qualitative and quantitative representations.

In the combinative tasks we used three types of activity contents: manipulative tasks with coloured sticks and plates, figural tasks with small printed images, and formal tasks with letters and symbols. Pupils were always asked to form and enumerate of all possible compositions. The tasks systematically contained the different combinatorial structures (variations, combinations, permutations, etc.), and they differed both in the number of initial elements that can be used for the formation of constructions and in the length of the constructions, and consequently in the number of all possible different constructions. With the three types of contents, our combinative tests contained 111 tasks (see Csapó, 1985a).

Let us consider the following task from the combinative test. The examinees have to form all of the possible combinations containing three letters, involving the letters

A, B, C, D or E. This is a task with a formal content. Manipulative and figural tasks are based on the same mathematical structure, but for purposes of illustration the formal content is the most suitable.

It is possible to enumerate the compositions in the following way:

(1) ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, CDE

The following enumeration also contains all of the different combinations, but in another sequence:

(2) ABC, BCD, CDE, DEA, EAB, ACE, CEB, EBD, BDA, DAC

Both (1) and (2) are perfect solutions of the task, but there are different ways of thinking behind them. Sequence (1) is an alphabetically ordered enumeration of the compositions, and it means that the subject who produces this enumeration has a system before he or she begins to solve the task. Sequence (2) illustrates an example of when the subject has found an idea to construct some of the compositions, but not all of them. This idea is to eliminate the first letter and add a new one at the other end of the composition. This algorithm produces only 5 compositions, but the subject them made a slight modification to the principle, and so the remaining 5 compositions could also be produced.

It is possible that the subject does not realise that in the case of combinations the sequence of elements in a composition does not mean a difference, e.g. ABC, ACB, BAC, etc. mean the same combination. Accordingly, some compositions may appear twice (or more times) in an enumeration. For example:

(3) ABC, ABD, ABE, ACD, ACE, ADE, ACB, ADB, BCD, BCE, BDE, CDE

Of course, the most typical form of error is that the subjects do not construct all the possible compositions. Either they apply a principle which does not produce all the different cases, or they do not have any system at all. For example:

(4) ABC, CDE, ADC, ADE, ABD, BCE, BCD, BDE

These enumerations can be regarded as printouts of the thinking, and by analysing them we can reconstruct the thinking strategies and the algorithms behind them, and hence we can identify the developmental level of thinking.

This very detailed representation needs a large information-storage capacity. For example, in our comprehensive examination we had ca. 900 subjects, each of whom had to solve all 111 combinative tasks. This means almost 100,000 task-solutions. If we wish to represent these in the above manner, this needs ca. 30 characters/task (there were shorter and longer enumerations, but 30 can be taken as the average), and so we should store almost 3 million characters. Fortunately, we can make the information more compact. Each composition can be designated by a number and then the enumerations can be represented by vectors.

In the case of the example presented above, we can regard the alphabetical sequence as a basis and denote the compositions as follows:

ABC = 1, ABD = 2, ABE = 3, ..., CDE = 10

Here the equivalent compositions (differing only in the sequence of elements within the compositions) are denoted by the same number. Thus, ACB, BAC, BCA, CAB and CBA are all denoted by 1, etc. In this way we have to use only one-third of the previous storage capacity, without losing any psychologically relevant information. (Of course, certain information is lost, which would be of importance in a more detailed analysis, but in this case it is worth paying this price for the more compact representation.) Whith the aim of easier processing, we always used the same length of vectors in a task; the missing compositions are denoted by 0. Furthermore, we used longer vectors than the total number of different compositions in order to permit the representation of surplus (repeated) compositions. We applied two additional places for these.

With this coding system, the above examples can be represented by the following code vectors:

(1') (1,2,3,4,5,6,7,8,9,10,0,0)
(2') (1,7,10,6,3,5,8,9,2,4,0,0)
(3') (1,2,3,4,5,6,1,2,7,8,9,10)
(4') (1,10,4,6,2,8,7,9,0,0,0,0)

For the quantitative analysis, we developed different measures to characterize the achievements. The easiest way is the dichotomous classification. The solution is good if the examinee enumerates all the necessary compositions and does not construct the same or equivalent compositions twice (or more times). And the solution is bad if not all the compositions are enumerated and/or some of them are given twice (or more times). Good solutions are scored by 1, and bad ones by 0. In this sense (1) and (2) are good solutions, while (3) and (4) are bad ones. The resulting score is denoted by d.

A more differentiated way to characterize the solutions is to count the number of the good compositions and the bad (surplus, repeated) compositions. The number of good compositions is denoted by g, and the number of bad ones by b.

The best way of quantification is, if the measure of achievement is a function of the complexity of the task, the number of good compositions and the number of bad ones enumerated in a solution. The complexity of the task can be represented by the total number of possible different compositions; this number is denoted by t. Accordingly, we can use the following formula as a quantitative measure (q) of the achievement in a task:

$q = (g(t-b))/t$, and if $b>t$ then $q=0$

The q ranges from 0 to t, and so it can be regarded as a weighted measure of achievement. If we wish to compare different tasks, it is better to use a comparable measure: the achievement as the percentage of the maximum. For this purpose, we can define q% in the following way:

$q\% = 100(q/t)$

We can compare the various ways of representation of the result of task solutions, as illustrated in the following table:

	Code vector	g	b	q	q%	d
(1'')	(1,2,3,4,5,6,7,8,9,10,0,0)	10	0	10	100	1
(2'')	(1,7,10,6,3,5,8,9,2,4,0,0)	10	0	10	100	1
(3'')	(1,2,3,4,5,6,1,2,7,8,9,10)	10	2	8	80	0
(4'')	(1,10,4,6,2,8,7,9,0,0,0,0)	8	8	8	80	0

As can be seen in this table, through the quantification we lose the possibility of making distinctions between psychologically important features. In the case of (1'') and (2''), each quantitative measure is the same, though the two code vectors represent two different ways of reasoning, or even two different developmental levels of thinking. In the case of (3'') and (4''), the q values are again equal, though the qualities of the solutions are very different.

On the other hand, the value d needs only 1 bit of memory capacity in the computer, q% needs 3 bytes, and the code vector 15-66 bytes, depending on the total number of different compositions.

3.2. Problems to solve by qualitative data analysis

At present we are using various methods of qualitative data analysis in our research, but in most cases the searching for and the adaptation of the appropriate method is still in progress. From the analyses that have been done, I shall present only one example, together with a number of unsolved problems, and further possibilities and requirements of qualitative data analysis.

3.2.1. Validating the developmental sequence by DEL-analysis

In our combinative tasks we use three types of content. In manipulative tasks, children have to construct compositions from coloured rods and enumerate all the possible solutions. With a figural content they have to indicate the compositions in small figures, and in the formal tasks they use letters and numbers. Figure 1 shows the enumerations of three tasks that have the same structure.

We examined the hypothesis that the sequence of the development is manipulative - figural - formal wia the results for 14-year-old pupils. In this task DEL=0.08 (n=550). We calculated the DEL values for the other 36 task-triplets in the same way. The highest DEL value was 0.30, and 26 from the 37 were below 0.2. (These results are summarized in Csapó, 1987.) Hence we do not have evidence that the development takes place in the manipulative - figural - formal sequence.

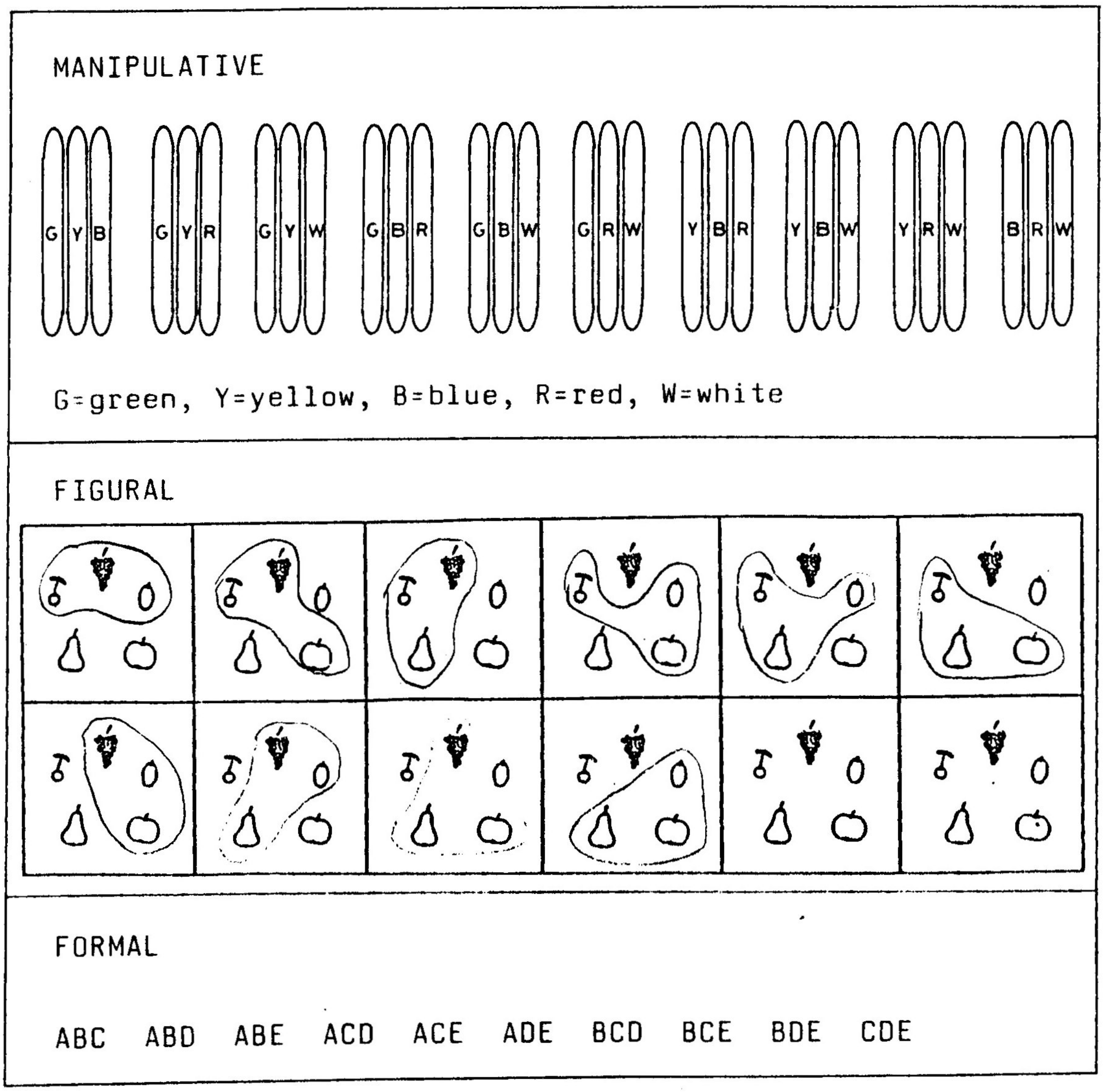


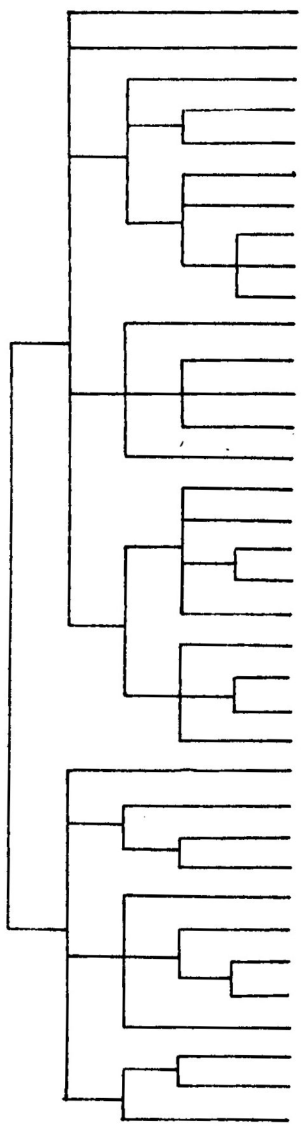
Figure 1
The enumerations in three combinatorial tasks with the same structure and with three types of content

3.2.2. Analysis of thinking strategies in combinatorial reasoning

As the representation of the qualitative entities requires much information, only a very simple example can be given here. One of the simplest combinatorial tasks is to devise all the possible variations of two red or blue rods repetitions being allowed. There are four different possibilities (colours are abbreviated with their initial letters):

RR, RB, BR, BB.

Of course, the sequence RR, BB, BR, RB, for instance, is also a good one, but with another manner of thinking behind it. There are 24 correct sequences. However, if the imperfect solutions are also taken into account, and we reserve two more places for the surplus compositions (we use a six-digit code) there are 15625 possible responses.



Code vector	Enumeration	Frequency	%
100000	RR	1	0.2
110000	RR,RR	1	0.2
120000	RR,RB,	1	0.2
123100	RR,RB,BR,RR	2	0.4
123400	RR,RB,BR,BB	16	2.8
124000	RR,RB,RR	4	0.7
124100	RR,RB,BB,RR	1	0.2
124300	RR,RB,BB,BR	35	6.3
124320	RR,RB,BB,BR,RB	1	0.2
124340	RR,RB,BB,BR,BB	1	0.2
130000	RR,BR	1	0.2
132000	RR,BR,RB	1	0.2
132300	RR,BR,RB,BR	1	0.2
132400	RR,BR,RB,RR	3	0.5
134200	RR,BR,BB,RB	6	1.1
142000	RR,BB,RB	7	1.3
142400	RR,BB,RB,RR	1	0.2
142300	RR,BB,RB,BR	163	29.4
142320	RR,BB,RB,RR,RB	1	0.2
142400	RR,BB,RB,RR	1	0.2
143000	RR,BB,BR	10	1.8
143200	RR,BB,BR,RB	76	13.7
143232	RR,BB,BR,RB,BR,RB	1	0.2
143300	RR,BB,BR,BR	1	0.2
200000	RB	1	0.2
213400	RB,RR,BR,BB	16	2.8
214000	RB,BB,BB	3	0.5
214300	RB,RR,BB,BR	12	2.2
230000	RB,BR	6	1.1
231000	RB,BR,RR	2	0.4
231400	RB,BR,RR,BB	131	23.6
231432	RB,BR,RR,BB,BR,RB	1	0.2
234100	RB,BR,BB,RR	37	6.7
241000	RB,BB,RR	2	0.4
241300	RB,BB,RR,BR	5	0.9
243100	RB,BB,BR,RR	2	0.4

Figure 2
Frequency distribution of different solutions
in a combinatorial task

Among the sequences there are perfectly equivalent pairs, which represent exactly the same way of reasoning. As there is no difference between the two colours from our point of view, the following two enumerations represent the same thinking strategy: RB, RR, BB, BR and BR, BB, RR, RB. If we substitute B by R and R by B in the second enumeration, we get the first one. This is an operation that can easily be done by a computer, and thus the number of possible code vectors will be half of the original number. Since we do not lose any relevant information by this transformation, in the enumerations that began with B, the letters were interchanged.

Figure 2 shows the frequency distribution of the solutions of 14-year-old pupils. The sequences that do not appear in the population are not displayed in the Figure. Accordingly, 36 different sequences remain. It can be seen on the Figure that the different code vectors do not occur with the same probability: there are some very characteristic sequences. In this simple example it is not obvious at first glance, that certain strategies are more effective than the other ones. However, if we have to construct compositions from more elements, we can produce all of the different posibilities only if we have a system to enumerate them. Only a system that determines the place of each composition in the sequence of the enumeration ensures that we produce each of them and do not produce any of them twice. It can be observed in Figure 2 that the preferred ways of enumeration are not the most effective ones. In the most frequent enumerations, pupils separate the compositions that contain the same elements: RR,BB,RB,BR=29.4%; RB,BR,RR,BB=23.6%; RR,BB,BR,RB=13.7%; RB,BR,BB,RR=6.7%. This strategy does not work in the more complicated cases. For example, if children have to compose three-letter compositions from the letters A and B, after composing AAA and BBB they run into difficulties, and compose other possibilities only by chance. The enumeration RR,RB,BR,BB, which was produced by only 2.8% of the pupils, has a more general algorithm behind it, which works well with more letters as well.

These qualitative analyses make it possible for us to compare our results with those of the Piagetian studies as well. We were able to identify the systems and parts of systems (successive pairings, juxtapositions, symmetries, etc.) described by Piaget, but the general picture emerging from our results is quite different from that of Piaget (Piaget and Inhelder, 1951). The Piagetian descriptions are usually not the most frequent ones statistically. We found that ca. 50% of the subjects reach the various developmental stages much earlier than in the Piaget studies, and ca. 30% do not reach the formal operational level till the age of 17. Thus, the age groups are very heterogeneous as concerns the level of their combinatorial reasoning.

In the most complicated tasks there are 15-20 compositions to devise and thousands of possible sequences of them. The preliminary analysis of the solutions reveals that every task involves 2-4 typical thinking strategies. Among these, there are usually a number of quite general and effective strategies.

3.2.3. Analysis of the system of combinative operations

From the abovementioned experiences we can form hypotheses for the qualitative analysis: if children use a type x strategy in (a simple) task a, then they will fail in (a more complicated) task b, and if they use a type y strategy in task a, then they will be successful in task b. This hypothesis can easily be tested, but before this we have to solve some other problems. We have to find general conditions for the computerized classification of the enumerations according to the thinking strategies (as the above four enumerations represent the same way of thinking).

In the more complex tasks, where pupils have to enumerate 12-16 compositions, it can happen that there is a very characteristic strategy behind the attempt, but a few compositions are wrong. We can recognize the strategy despite the fault, but the computer misclassifies them and we get too many different classes. Therefore, we first have to instruct the computer to identify the strategies, even if there are mistakes in the enumerations.

Consequently, our current efforts are concentrated on developing more sophisticated computer programs that are able to identify the thinking strategies even if there are mistakes in the enumerations, and to classify pupils into groups according to the qualitative differences of thinking. In this way we can examine relationships between certain qualitative characteristics of the thinking and affective or environmental variables. For example, we can analyse what differences there are in the social background and school achievements of the different groups formed on the basis of the qualitative characteristics of reasoning.

4. Representation of the quality of propositional reasoning

4.1. Tests, tasks and qualitative data

In the examinations of logical ability, we devised tests for the study of the binary operations of propositional logic, for the most frequent schemes of inferences and for more complex logical structures. In this paper I shall present only a few examples of the easiest tasks and the representation of the quality of different solutions.

To illustrate the tasks of the logical ability tests, let us consider the following example:

You will find here a sentence spoken by Peter and below it a series of true statements. These statements are denoted by letters A, B, C and D. Read the statements carefully and decide in which cases Peter's sentence is true, and in which cases it is false. In the cases when Peter's sentence is true, put a circle round the letter before the statement, e.g. A. If Peter's sentence is not true, cross out the letter, e.g. ~~A~~.

Peter says:
THIS AFTERNOON I SHALL WATCH TV AND PLAY WITH MY CAR.

Statements:
A. He will watch TV, and he will play with his car.
B. He will watch TV, and he will not play with his car.
C. He will not watch TV, and he will play with his car.
D. He will not watch TV, and he will not play with his car.

This task (like many others in our tests) consists of two parts: a complex proposition containing a logical operation and a series of statements. The statements correspond to the truth table of the logical operation. In this example the operation is the conjunction (p and q), one of the most frequently used binary operations of propositional logic. Thus, (according to the convention of formal logic) this complex proposition is true if and only if both propositions in it are true. Accordinly, the solution is perfect if the examinee declares A to be true, and B, C and D to be false.

The examinee's response can be denoted by 0 or 1, and the perfect solution by the vector (1000). This vector can be regarded as a binary number and can be converted into a decimal number. The decimal equivalent of 1000 is 8. Thus, the coding process is:

Response pattern	Notation	Binary code	Decimal code
A	1		
B	0	1000	8
C	0		
D	0		

Altogether there are 16 possible different response patterns: 0000, 0001, ..., 1111; or in decimal form 0, 1, ..., 15. If the examinee has no idea about the solution and answers randomly, the probability of any one pattern is 1/16 (6.25%).

The examinee's answers can be evaluated in different ways. We could use here the dichotomous scoring: the examinees receive a score of 1 if they produce the good response pattern (all of the four decisions are right); otherwise their score is 0. This score is denoted by d.

It is also possible to quantify the achievements according to how many correct decisions there are in the response pattern. Hence, the perfect solution means a score of 4. This quantitative measure is denoted by q. Similarly to the case of combinative operations, it is also possible to use comparable measures of the achievements (q%). This can be defined in the same way as with combinative operations:

$$q\% = 100(q/t)$$

where t is the number of decisions in the task (in the case of binary operations, t=4).

Let us compare the qualitative and quantitative representations of some solutions. Let us suppose that four pupils produce the following response patterns: 1000, 1110, 1101, 1001. The different measures of these responses are as follows:

	Code vector	q	q%	d
(1)	(1000)	4	100	1
(2)	(1110)	2	50	0
(3)	(1101)	2	50	0
(4)	(1001)	3	75	0

At this point we have to consider that any of the 16 possible response patterns represents one of the 16 binary operations of propositional logic. Therefore, the response pattern can be regarded as a printout of how the examinee understands the complex proposition, and the respective operations can be reconstructed from them. Thus, pattern (1) is the right solution, as it represents a conjunction; the other three indicate misunderstandings, as (2) represents a disjunction, (3) a conditional and (4) a biconditional.

From this point of view, only the code vectors reflect the real nature of the examinees' thinking. The dichotomous scoring is a correct means of quantification, but in the case of the operations of propositional logic the above-defined q and q% (which otherwise seem to be rather evident parameters) could provide false results: we can not say that to perceive a conjunction as a biconditional is a better achievement than as to consider it as a disjunction or as a conditional.

These examples relate only to the representation of a single solution by a single pupil. However, in our tests of propositional logic we have 10 different task structures, and all of these structures are formed into tasks by using three different contents.

By applicaion of the relation of partial ordering to the code vector of their truth table, the 16 structures can be ordered into a lattice structure. This lattice structure of the binary operations of propositional logic has a central role in the Piagetian theory: it organizes the logical operations of children into a whole and the formal level of thinking can be characterized by its appearance (see Inhelder and Piaget, 1955, and for a more recent discussion of the problem Ascher, 1984. The same algebraic structure in Rudinger et al., 1985, in another context represents a hypothesis and is examined by DEL-analysis). This lattice structure is displayed in Figure 3.

Of the 16 possible operations we used only 10; the other six are: the null operation (or contradiction); the tautology; and four other operations that give the first or the second operation or the negation of them independently of the other one. The codes of these six omitted operations are:

0000, 1111, 1100, 1010, 0101, 0011. As can be seen in the Figure 3, these codes are situated symmetrically in the lattice.

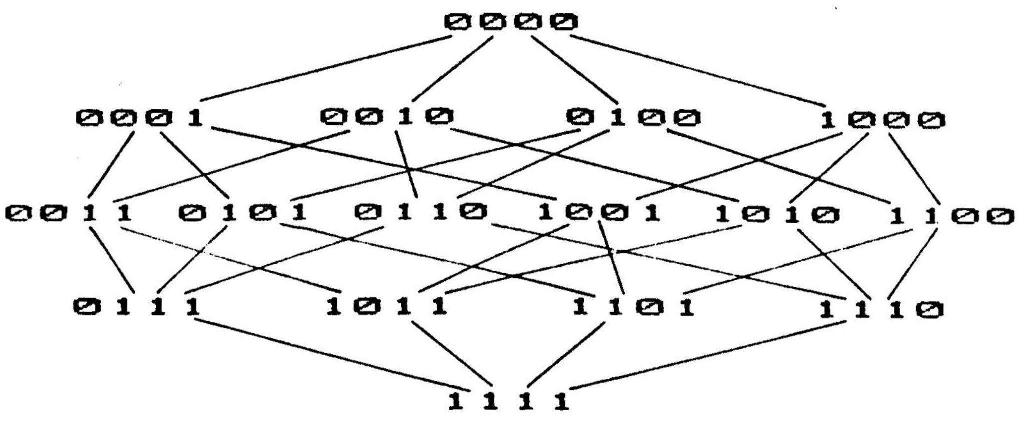


Figure 3
Lattice structure of the 16 binary operations

If we wish to represent the structure of the propositional operations of a given child, we can display the correct answers and the misinterpretations of the complex propositions involving the respective operations in the lattice. Figure 4 shows the structure of the propositional logic of a 14-year-old child. The arrows point from the operations that really connect the propositions (there are frames around them) to the one, that the child perceives in the complex proposition.

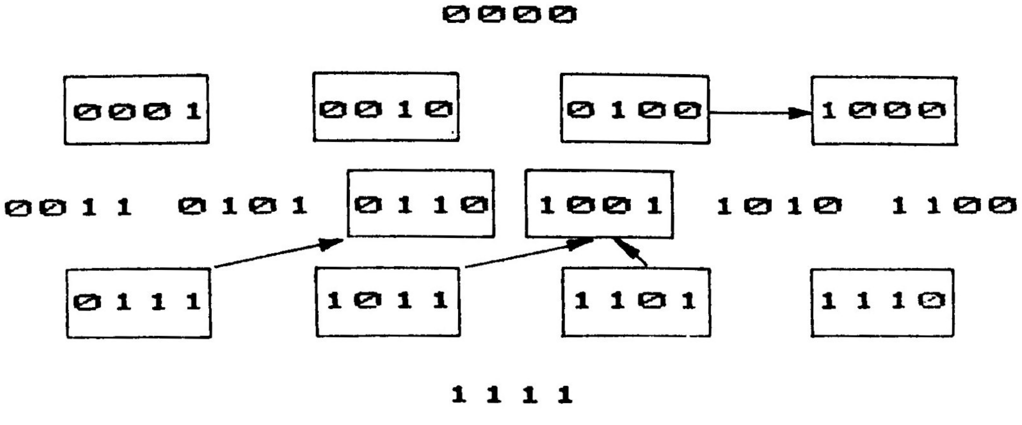


Figure 4
Representation of a child's structure of propositional logic

The above examples illustrate only the simplest tasks of the tests of logic. More complex logical structures can be examined in the same way. In a complex sentence we can combine two or more operations. The following example relates to the combinations of two operations. In this sentence there are a disjunction and a Peirce operation. Its structure is: 'neither (p or q), nor r'.

You will find here a sentence spoken by Jimmy and below it a series of true statements. These statements are denoted by the letters A, B, ... H. Read the statements carefully and decide in which cases Jimmy's sentence is true, and in which cases it is false. In the cases when Jimmy's sentence is true, put a circle round the letter before the statement, e.g. A. If Jimmy's sentence is not true, cross out the letter, e.g. ~~A~~.

Jimmy says:
I SHALL NEITHER PLAY WITH MY CAR OR WITH MY PLANE, NOR WATCH TV.

Statements:

A. He will play with his car, he will play with his plane, and he will watch TV.
B. He will play with his car, he will play with his plane, and he will not watch TV.
C. He will play with his car, he will not play with his plane, and he will watch TV.
D. He will play with his car, he will not play with his plane, and he will not watch TV.
E. He will not play with his car, he will play with his plane, and he will watch TV.
F. He will not play with his car, he will play with his plane, and he will not watch TV.
G. He will not play with his car, he will not play with his plane, and he will watch TV.
H. He will not play with his car, he will not play with his plane, and he will not watch TV.

In this task there are 256 different possible response patterns, and thus the probability of random choice is 0.0039. According to the convention of formal logic, the complex sentence is true only if p and q and r are all false, so the perfect solution can be represented by the vector (00000001).

4.2. Analysis of the structures of propositional logic

The qualitative data analysis can be used to test several hypotheses in the study of children's structures of logical operations. I shall give here only three possibilities: (1) examination of the individual's logical structures and developmental levels, (2) study of the effect of different contents of tasks, and (3) analysis of the developmental tendencies of individuals and populations.

4.2.1. Analysis of an individual's logical structures

When a certain response pattern appears, it can have three causes: (1) the examinee has a working operational system and he or she gives the perfect solution; (2) the complex sentence has a certain meaning for him or her, but not the meaning, which is corect according to formal logic; (3) the examinee does not understand the task, and/or does not have a working operational system, and consequently merely guesses. From the performance of a single person in a single task, we can not decide to which of the above classes he or she belongs. However, if we analyse the responses of the same person in different tasks (the whole set of response patterns), we can infer with a high degree of probability to which of the above three groups he or she belongs in a certain task. As illustrated in Figure 2, it is possible to characterize the pupils with a graph (or a system of relations). By analysing these graphs, we can determine the most typical ones, and set up rules to classify pupils into categories according to their operational structure.

This classification can be used for the diagnosis of their development, and can be applied as a feed-back in the educational processes. This structural analysis may result in a better basis for the evaluation than the conventional scores. The classification provides categorial data that can be used in other qualitative analyses.

4.2.2. Validating and improving the tasks and tests

The effects of the contents of tasks can also be studied, as we have tasks with three different contents for 10 operations of propositional logic. One of the easiest methods is to examine whether tasks with the same structure but with different contents yield the same structures for thr pupils' response patterns. The hypothesis that all three tasks are equivalent can be tested by DEL-analysis. In this way, children who respond in the same way in all three tasks will be classified into the "permissible" cells, and the others into the "error" cells.

This method of validating requires an exact mapping between the response patterns of the tasks with different contents, and it can give an extensive feed-back for the improvement of tasks and the validity of tests. There is a less strict way of validating tests by qualitative analysis: to examine the consistency of the classification of pupils given by the three tests.

4.2.3. Qualitative differences within a population

One of the most interesting problems of our research is the examination of the operational structures of a population and their changes over time. For the representation of the distribution of response patterns in the population, an example is given in Table 1. The Table lists the codes of the response patterns, the explanation of the responses, and the respective operations that can be reconstructed from these explanations. (It must be mentioned that, as the formulas of logic can be transcribed into one another, the operations displayed in the Table are only possible interpretations.)

Table 1 Frequency distribution of response patterns in the task 'p or q' at the age of 14 (n=577)

Bin. code	Dec. code	Explanation	Operation	Frequency	%
0000	0		contradiction	1	.2
0001	1	$(\bar{p}\&\bar{q})$	neither p nor q	3	.5
0010	2	$(\bar{p}\&q)$	not(if q then p)	16	2.8
0011	3	$(\bar{p}\&q)$or$(\bar{p}\&\bar{q})$	$\bar{p}$	2	.3
0100	4	$(p\&\bar{q})$	not(if p then q)	7	1.2
0101	5	$(p\&\bar{q})$or$(\bar{p}\&\bar{q})$	$\bar{q}$	2	.3
0110	6	$(p\&\bar{q})$or$(\bar{p}\&q)$	either only p or only q	5	.9
0111	7	$(p\&\bar{q})$or$(\bar{p}\&q)$or$(\bar{p}\&\bar{q})$	p or q or neither of them	0	.0
1000	8	$(p\&q)$	p and q	161	27.9
1001	9	$(p\&q)$or$(\bar{p}\&\bar{q})$	p if and only if q	4	.7
1010	10	$(p\&q)$or$(\bar{p}\&q)$	q	7	2.8
1011	11	$(p\&q)$or$(\bar{p}\&q)$or$(\bar{p}\&\bar{q})$	if q then p	0	.0
1100	12	$(p\&q)$or$(p\&\bar{q})$	p	21	3.6
1101	13	$(p\&q)$or$(p\&\bar{q})$or$(\bar{p}\&\bar{q})$	if p then q	0	.0
1110	14	$(p\&q)$or$(p\&\bar{q})$or$(\bar{p}\&q)$	p or q	347	60.0
1111	15	$(p\&q)$or$(p\&\bar{q})$or$(\bar{p}\&q)$or$(\bar{p}\&\bar{q})$	tautology	1	.2

As the data show, only 60.0% of children give the right solution. It is also very interesting to examine, what the typical misunderstandings are. This can be done more easily if we display our data in the form of a lattice structure, as can be seen in Figure 5. The correct response pattern, which relates to the disjunction (1110), is indicated by a frame. The numerical data above the codes are the rates of the respective response patterns (in %).

As the Figure shows, the tendencies of the misinterpretations are very characteristic. One of the most interesting facts is that the directions of the misinterpretations point to operations that have less "true" value in their truth table. The most frequent error is to perceive a conjunction in the complex sentence insted of a disjunction (what it really is).

0.2
0000

0.5	2.8	1.2	27.9
0001	0010	0100	1000

0.3	0.3	0.9	0.7	2.8	3.6
0011	0101	0110	1001	1010	1100

0	0	0	60.0
0111	1011	1101	1110

0.2
1111

Figure 5
Response structure of 14-year-old children
in the task 'p or q'

We calculated the frequency distributions in the same way for all tasks with all the three structures. Hence it is possible to form and test hypotheses connecting more operations.

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